Formula sheet for Power Electronics

Feel free to distribute it to your students. Please address any mistakes to hadeedahmedsher@gmail.com . Latex template by divisbyzero.com

Basic expressions

Average expression

$$X_{avg} = \frac{1}{T}\int_0^t x(t)dt \tag{1}$$

RMS expression

$$X_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2(t)dt} \tag{2}$$

$$X_{rms} = \sqrt{\sum_{n=1...}^{N} X_{n,rms}^2} \tag{3}$$

Fourier series

$$f(t) = a_0 + \sum_{n=1}^{\infty}[a_n cos(n\omega_0 t) + b_n sin(n\omega_0 t)] \tag{4}$$

where,

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)dt \tag{5}$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)cos(n\omega_0 t)dt \tag{6}$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)sin(n\omega_0 t)dt \tag{7}$$

Power and Energy

$$Instantaneous Power p(t) = v(t)i(t) \tag{8}$$

$$Power(P) = \sum_{n=1}^{\infty} P_n = \sum_{n=1}^{\infty} I_{n,rms}^2 R \tag{9}$$

$$Averagepower P = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)dt = \frac{W}{T} \tag{10}$$

$$Averagepower P_{dc} = V_{dc}I_{avg} \tag{11}$$

$$Apparent Power S = V_{rms}I_{rms} \tag{12}$$

$$S = \sqrt{3}V_{L-L,rms}I_{S,rms} \tag{13}$$

$$Reactivepower Q = V_{rms}I_{rms}sin(\theta - \phi) \tag{14}$$

$$Energy(W) = \int_{t_1}^{t_2} p(t)dt \tag{15}$$

$$W = \int_0^T p(t)dt \tag{16}$$

$$Powerfactor(pf) = \frac{P}{S} \tag{17}$$

Inductor

$$Voltage \implies v_L = L\frac{di}{dt} \tag{18}$$

$$Current i_L = \frac{1}{L}\int v_L(dt) + I_{0-} \tag{19}$$

$$Energy w(t) = \frac{1}{2}Li^2(t) \tag{20}$$

$$\tau = \frac{L}{R} \tag{21}$$

Capacitor

$$Current i_C = C\frac{dv}{dt} \tag{22}$$

$$Voltage v_c = \frac{1}{C}\int i_C(dt) + V_{0-} \tag{23}$$

$$Energy w(t) = \frac{1}{2}Cv^2(t) \tag{24}$$

$$\tau = \frac{1}{RC} \tag{25}$$

Phasors

$$Impedance(Z) = \sqrt{(R)^2 + (\omega L)^2} \tag{26}$$

$$\theta = tan^{-1}\left(\frac{\omega L}{R}\right) \tag{27}$$

Half wave rectifier with RL load

$$i_{(\omega t)} = \begin{cases} \frac{V_m}{Z}\left[sin(\omega t - \theta) + sin(\theta)e^{-\omega t/\omega\tau}\right] & \text{for } 0 \le \omega t \le \beta \\ 0 & \beta \le \omega t \le 2\pi \end{cases} \tag{28}$$

$$V_o = \frac{1}{2\pi}\int_\alpha^\beta V_m sin(\omega t)d(\omega t) = \frac{V_m}{2\pi}(cos\alpha - cos\beta) \tag{29}$$

Average current (RL series load without freewheeling diode)

$$I_o = \frac{1}{2\pi}\int_0^\beta i(\omega t)d(\omega t) \tag{30}$$

With free wheeling diode

$$v(t) = \frac{V_m}{\pi} + \frac{V_m}{2}sin(\omega_o t) - \sum_{n=2,4,6...}^{\infty}\frac{2V_m}{(n^2-1)\pi}cos(n\omega_o t) \tag{31}$$

With source inductance

$$V_o = \frac{V_m}{2\pi}(1 + cosu) = \frac{V_m}{\pi}\left(1 - \frac{I_L X_s}{2V_m}\right) \tag{32}$$

Half wave rectifier with RL source load

$$i_o(\omega t) = \begin{cases} \frac{V_m}{Z}sin(\omega t - \theta) - \frac{V_{dc}}{R} + Ae^{-\omega t/\omega\tau} & \text{for } \alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases} \tag{33}$$

Half wave rectifier with RC parallel load

$$v_o(\omega t) = \begin{cases} V_m sin(\omega t) & \text{Diode on} \\ V_\theta e^{-(\omega t - \theta)/\omega RC} & \text{Diode off} \end{cases} \tag{34}$$

capacitor current

$$i_c(\omega t) = \begin{cases} -\left(\frac{V_m sin\theta}{R}\right)e^{-(\omega t - \theta)/\omega RC} & \text{Diode off} \\ \omega C V_m cos(\omega t) & \text{Diode on} \end{cases} \tag{35}$$

The peak to peak ripple

$$\Delta V_o = V_m - V_m sin\alpha = V_m(1 - sin\alpha) \tag{36}$$

$$\Delta V_o \approx \frac{V_m}{fRC} \tag{37}$$

Peak diode current

$$I_{D,peak} = V_m\left(\omega C cos\alpha + \frac{sin\alpha}{R}\right) \tag{38}$$

Half wave controlled rectifier with R load

$$V_o = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m sin(\omega t)d(\omega t) = \frac{V_m}{2\pi}(1+cos\alpha) \tag{39}$$

Half wave controlled rectifier with RL load

$$i_{(\omega t)} = \begin{cases} \frac{V_m}{Z}\left[sin(\omega t-\theta) - sin(\alpha-\theta)e^{(\alpha-\omega t)/\omega\tau}\right] & \text{for } \alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases} \tag{40}$$

Full wave rectifier

Full wave rectified waveform Fourier is

$$v_o(t) = \frac{2V_m}{\pi} + \sum_{n=2,4...}^{\infty}\left(\frac{2V_m}{\pi}\left(\frac{1}{n-1} - \frac{1}{n+1}\right)\right)cos(n\omega_o t+\pi) \tag{41}$$

Ripple voltage for C filter

$$\Delta V_o \approx \frac{V_m\pi}{\omega RC} = \frac{V_m}{2fRC} \tag{42}$$

With LC filter
In CCM

$$V_o = \frac{2V_m}{\pi} \tag{43}$$

$$I_L = \frac{2V_m}{\pi R} \tag{44}$$

$$\frac{3\omega L}{R} > 1 \tag{45}$$

In DCM average inductor current.

$$i_L = \frac{1}{\pi}\int_{\alpha}^{\beta}\frac{1}{\omega L}\left[V_m(cos\alpha - cos\omega t) - V_o(\omega t-\alpha)\right]d(\omega t) \tag{46}$$

RLE load CCM

$$I_o = \frac{V_o - E}{R} = \frac{\frac{2V_m}{\pi} - E}{R} \tag{47}$$

$$v_o(t) = V_o + \sum_{n=2,4..}^{\infty} V_n cos(n\omega_0 t+\pi) \tag{48}$$

where, $VC_o = \frac{2V_m}{\pi}$ and $V_n = \frac{2V_m}{\pi}\left(\frac{1}{n-1} - \frac{1}{n+1}\right)$
Full controlled with RL load
DCM

$$\begin{aligned} i_o(\omega t) = \frac{V_m}{Z}[sin(\omega t-\theta) - sin(\alpha-\theta) \\ e^{-(\omega t-\alpha)/\omega\tau}]for\alpha \le \omega t \le \beta \end{aligned} \tag{49}$$

$$\beta < \alpha + \pi \implies DCM \tag{50}$$

For CCM

$$\alpha \le tan^{-1}\left(\frac{\omega L}{R}\right) \tag{51}$$

$$v_o(\omega t) = V_0 + \sum_{n=1}^{\infty} V_n cos(n\omega_0 t+\theta_n) \tag{52}$$

$$V_o = \frac{1}{\pi}\int_{\alpha}^{\alpha+\pi} V_m sin(\omega t)d(\omega t) = \frac{2V_m}{\pi}cos\alpha \tag{53}$$

The harmonics are

$$V_n = \sqrt{a_n^2 + b_n^2} \tag{54}$$

where,

$$a_n = \frac{2V_m}{\pi}\left[\frac{cos(n+1)\alpha}{n+1} - \frac{cos(n-1)\alpha}{n-1}\right] \tag{55}$$

$$b_n = \frac{2V_m}{\pi}\left[\frac{sin(n+1)\alpha}{n+1} - \frac{sin(n-1)\alpha}{n-1}\right] \tag{56}$$

$$V_o = \frac{2V_m}{\pi}cos\alpha \tag{57}$$

DC-AC inverters

Square wave inverter(RL series load)

$$i_o(t) = \begin{cases} \frac{V_{dc}}{R} + \left(I_{min} - \frac{V_{dc}}{R}\right)e^{-t/\tau} & \text{for } 0 \le\ t \le \frac{T}{2} \\ \frac{-V_{dc}}{R} + \left(I_{max} + \frac{V_{dc}}{R}\right)e^{-(t-T/2)/\tau} & \text{for } \frac{T}{2} \le\ t \le T \end{cases} \tag{58}$$

$$I_{max} = -I_{min} = \frac{V_{dc}}{R}\left(\frac{1-e^{-(T/2\tau)}}{1+e^{-(T/2\tau)}}\right) \tag{59}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)d(t)} = \sqrt{\frac{2}{T}\int_0^{T/2}\left[\frac{V_{dc}}{R} + \left(I_{min} - \frac{V_{dc}}{R}\right)e^{-t/\tau}\right]^2 dt} \tag{60}$$

$$v_o(t) = \sum_{1,3,5..}^{\infty}\frac{4V_{dc}}{n\pi}sinn\omega_0 t \tag{61}$$

$$THD = \frac{\sqrt{\sum_{n=2}^{\infty}(V_{n,rms})^2}}{V_{1,rms}} = \frac{\sqrt{V_{rms}^2 - V_{1,rms}^2}}{V_{1,rms}} \tag{62}$$

Quasi Square wave inverter(RL series load)

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi-\alpha} V_{dc}^2 d(\omega t)} = V_{dc}\sqrt{1-\frac{2\alpha}{\pi}} \tag{63}$$

$$v_n = \frac{2}{\pi}\int_{\alpha}^{\pi-\alpha} V_{dc} sin(n\omega_0 t)d(\omega_0 t) = \frac{4V_{dc}}{n\pi}cos(n\alpha) \tag{64}$$

$$V_1 = \left(\frac{4V_{dc}}{\pi}\right)cos\alpha \tag{65}$$

PWM inverter

$$m_f = \frac{f_{carrier}}{f_{reference}} = \frac{f_{tri}}{f_{sine}} \tag{66}$$

$$m_a = \frac{V_{m,reference}}{V_{m,carrier}} = \frac{V_{m,sine}}{V_{m,tri}} \tag{67}$$

Bipolar PWM inverter

$$v_o(t) = \sum_{n=1}^{\infty} V_n sin(n\omega_0)t \tag{68}$$

6 step inverter

$$V_{n,L-L} = \left|\frac{2V_{dc}}{3n\pi}\left[2 + cos\left(\frac{n\pi}{3}\right) - cos\left(\frac{n2\pi}{3}\right)\right]\right| \ for \quad n = 1,5,7,11,13... \tag{69}$$

3 phase PWM inverter

$$V_{n3} = \sqrt{A_{n3}^2 + B_{n3}^2} \tag{70}$$

where,

$$\begin{cases} A_{n3} = V_n sin\left(\frac{n\pi}{2}\right)sin\left(\frac{n\pi}{3}\right) \\ B_{n3} = V_n cos\left(\frac{n\pi}{2}\right)sin\left(\frac{n\pi}{3}\right) \end{cases} \tag{71}$$

DC-DC Converters

Buck converter

$$V_o = V_s D \tag{72}$$

$$I_{max} = I_L + \frac{\Delta i_L}{2} = \frac{V_o}{R} + \frac{1}{2}\left[\frac{V_o}{L}(1-D)T\right] = V_o\left(\frac{1}{R} + \frac{1-D}{2Lf}\right) \tag{73}$$

$$L_{min} = \frac{(1-D)R}{2f} \tag{74}$$

$$L = \left(\frac{V_s - V_o}{\Delta i_L f}\right)D = \frac{V_o(1-D)}{\Delta i_L f} \tag{75}$$

$$C = \frac{1-D}{8L(\Delta V_o/V_o)f^2} \tag{76}$$

In DCM,

$$\frac{V_o}{V_s} = \frac{2}{1+\sqrt{1+4K/D_1^2}} \tag{77}$$

where, $K = 2L/RT_s$.
The boundary between CCM and DCM occurs when $D_1 = 1 - D$

Boost converter

$$V_o = \frac{V_s}{1-D} \tag{78}$$

$$I_{max} = I_L + \frac{\Delta i_L}{2} = \frac{V_s}{(1-D)^2 R} + \frac{V_s DT}{2L} \tag{79}$$

$$I_{min} = I_L - \frac{\Delta i_L}{2} = \frac{V_s}{(1-D)^2 R} - \frac{V_s DT}{2L} \tag{80}$$

$$L_{min} = \frac{D(1-D)^2 R}{2f} \tag{81}$$

$$L = \frac{V_s DT}{\Delta i_L} = \frac{V_s D}{\Delta i_L f} \tag{82}$$

$$C = \frac{D}{R(\Delta V_o/V_o)f} \tag{83}$$

Buck-Boost Converter

$$V_o = -V_s\left(\frac{D}{1-D}\right) \tag{84}$$

$$I_{max} = I_L + \frac{\Delta i_L}{2} = \frac{V_s D}{R(1-D)^2} + \frac{V_s DT}{2L} \tag{85}$$

Minimum inductor current is

$$I_{min} = I_L - \frac{\Delta i_L}{2} = \frac{V_s D}{R(1-D)^2} - \frac{V_s DT}{2L} \tag{86}$$

$$L_{min} = \frac{(1-D)^2 R}{2f} \tag{87}$$

$$L = \frac{V_s DT}{\Delta i_L} = \frac{V_s D}{f \Delta i_L} \tag{88}$$

$$C = \frac{D}{R(\Delta V_o/V_o)f} \tag{89}$$

Cuk Converter

$$V_o = \frac{-D}{D'}V_s \tag{90}$$

$$L_1 = \frac{V_s D}{f\Delta i_1} \tag{91}$$

$$L_2 = D\frac{V_1 + V_2}{f\Delta i_2} \tag{92}$$

$$C_1 = \frac{V_o D}{Rf\Delta v_{c_1}} \tag{93}$$

$$C_2 = \frac{1-D}{(\Delta V_o/V_o)8L_2 f^2} \tag{94}$$

SEPIC Converter

$$V_o = V_s\left(\frac{D}{D'}\right) \tag{95}$$

$$C_o = \frac{D}{R(\Delta V_o/V_o)f} \tag{96}$$

$$C_1 = \frac{D}{R(\Delta V_{c1}/V_o)f} \tag{97}$$

Flyback converter

$$V_o = V_s\left(\frac{D}{1-D}\right)\left(\frac{N_2}{N_1}\right) \tag{98}$$

$$\begin{cases} i_D = -i_1\left(\frac{N_1}{N_2}\right) = i_{Lm}\left(\frac{N_1}{N_2}\right) \\ v_{sw} = V_s - v_1 = V_s + V_o\left(\frac{N_1}{N_2}\right) \\ i_R = \frac{V_o}{R} \\ i_C = i_D - i_R = i_{Lm}\left(\frac{N_1}{N_2}\right) - \frac{V_o}{R} \end{cases} \tag{99}$$

$$L_m = \frac{V_s DT}{\Delta i_{Lm}} = \frac{V_s D}{f\Delta i_{Lm}} \tag{100}$$

$$(L_m)_{min} = \frac{(1-D)^2 R}{2f}\left(\frac{N_2}{N_1}\right)^2 \tag{101}$$

$$\frac{\Delta V_o}{V_o} = \frac{D}{RCf} \tag{102}$$

Forward converter

$$V_o = DV_s\left(\frac{N_2}{N_1}\right) \tag{103}$$

$$C = \frac{1-D}{8L_x(\Delta V_o/V_o)f^2} \tag{104}$$

Capacitor Resistance

$$\Delta V_{o,ESR} = \Delta i_c r_c \tag{105}$$

Heat Sinks

$$R_\theta = \frac{T_1 - T_2}{P} \tag{106}$$

$$T_J = P(R_{\theta,JC} + R_{\theta,CS} + R_{\theta,SA}) + T_A \tag{107}$$